

Sixty-six presents were announced as having been received since the last meeting, including, amongst others :—

Astrographic Chart ; 18 charts, presented by the Royal Observatory, Greenwich ; 13 presented by the Tacubaya Observatory, and 41 by the French Minister of Public Instruction, from the Algiers, Bordeaux, and Paris Observatories.

### *Practical Suggestions on Mathematical Notation and Printing.*

[Reprinted by permission from the *Proceedings of the Royal Society*, A., vol. lxxxii., 1909.]

It is a subject of common complaint that mathematical manuscripts are often prepared for press without due regard for the difficulties encountered in setting up the type, or for the appearance of the printed page.

The Council of the Royal Society have had under consideration for some time the desirability of taking steps with a view to diminish the expense of printing and proof-corrections, and to avoid waste of space, and undue variety of notation in papers by different authors in the same volume. They have approved of the reprinting, with modifications and additions, of the substance of a Report to the British Association on this subject, in the hope that greater uniformity and facility in mathematical typography may thereby be promoted. The recommendations which follow are now offered, not in any authoritative way, but simply as a consensus of opinion ; to this end it is understood that they were submitted in advance, for consideration and criticism, to the Council of the London Mathematical Society.

#### *Abstract of Report of British Association Committee.\**

With a view to the questions referred to them for consideration, the Committee appointed by the British Association made inquiries into the nature and processes of mathematical printing, and the difficulties attendant thereon ; and it appeared to them that a statement of the results of these inquiries would form the best introduction to the suggestions which they had to make.

The process of "composition" of ordinary matter consists in arranging types uniform in height and depth (or "body" as it is termed) in simple straight lines. The complications peculiar to mathematical matter are mainly of two kinds.

\* Report of the Committee, consisting of W. Spottiswoode, F.R.S., Prof. Stokes, F.R.S., Prof. Cayley, F.R.S., Prof. Clifford, F.R.S., and J. W. L. Glaisher, F.R.S., appointed to report on Mathematical Notation and Printing, with the view of leading mathematicians to prefer in optional cases such forms as are more easily put into type, and of promoting uniformity of notation.—*B. A. Report*, 1875, pp. 337–339.

First, figures or letters of a smaller size than those to which they are appended have to be set as indices or suffixes ; and consequently, except when the expressions are of such frequent occurrence as to make it worth while to have them cast upon type of the various bodies with which they are used, it becomes necessary to fit these smaller types in their proper positions by special methods. This process, which is called "justification," consists in filling up the difference between the bodies of the larger and smaller types with suitable pieces of metal.

The second difficulty arises from the use of lines or "rules" which occur between the numerator and denominator of fractions, and (in one mode of writing) over expressions contained under radical signs. In whatever part of a line such a rule is used, it is necessary to fill up, or compensate, the thickness of it throughout the entire line.

The complications above described may arise in combination or may be repeated more than once in a single expression ; and in proportion as the pieces to be "justified" become smaller and more numerous, so do the difficulties of the workman, the time occupied on the work, and the chances of subsequent dislocation of parts augment.

The cost of "composing" mathematical matter may now (1908) in general be estimated at somewhat more than twice that of ordinary or plain matter, the recent adoption of the point system in the casting of types having greatly simplified mathematical justification.

There are many expressions occurring in mathematics which are capable of being written in more than one way ; and of these some present much greater difficulties to the printer than others. This being so, the Committee were of opinion that instead of making any specific recommendations, the most useful course they could take would be to append a table of equivalent forms specifying those which do and those which do not involve justification, and also a list of mathematical signs which may fairly be expected to be found, in the usual sizes, ready to hand among a printer's materials.

In recommending in this qualified way some forms of notation in preference to others, the Committee wished it to be distinctly understood that they were speaking from the printing, and not from the scientific point of view ; and they were quite aware that, even if some of the easier forms should be adopted in some cases, they may still not be of universal application, and that there may be passages, memoirs, or even whole treatises in which they would be inadmissible.

The Committee drew attention to the advantages which may incidentally accrue to mathematical science by even a partial adoption of the modifications suggested. Anything which tends towards uniformity in notation may be said to tend towards a common language in mathematics ; and whatever contributes to cheapening the production of mathematical books must ultimately assist in disseminating a knowledge of the science of which they treat.

## MATHEMATICAL SIGNS NOT INVOLVING "JUSTIFICATION."

$$\begin{array}{ccccccccc} \times & - & + & = & \sqrt{} & \pm & :: & \dots & \therefore : \\ & & & & & & & & \} & < > \div \\ & & & & & & & & & \\ & ( & [ & } & \int & \sqrt{} & & & \\ & a & a' & a_1 & a^2 & a_2 & a^{\frac{1}{2}} & a_i & \end{array}$$

## EQUIVALENT FORMS.

Involving justification.

$$\begin{aligned} & \frac{x}{a} \\ & \sqrt{x} \\ & \sqrt[3]{x} \\ & \sqrt{x-y} \\ & \sqrt{-1} \\ & x \cdot \overline{x+a} \\ & \epsilon^{\frac{n\pi x}{a}} \end{aligned}$$

Not involving justification.

$$\begin{aligned} & x/a \text{ or } x \div a \text{ or } x : a \\ & \sqrt{x} \text{ or } x^{\frac{1}{2}} \\ & \sqrt[3]{x} \text{ or } x^{\frac{1}{3}} \\ & \sqrt{(x-y)} \text{ or } (x-y)^{\frac{1}{2}} \\ & i \text{ or } i \\ & x(x+a) \\ & e^{n\pi x/a} \end{aligned}$$

This British Association List, which has been abbreviated and modified, is now incorporated in the following :—

RECOMMENDATIONS REGARDING MATHEMATICAL NOTATION  
AND PRINTING.

Always—

instead of	$\frac{x}{3}$	$\frac{a+b}{2}$	$\frac{a+\frac{b}{2}}{\frac{c}{3}+\frac{d}{4}}$	$\frac{a}{b+\frac{c}{d}}$	$\sqrt{x}$	$\sqrt{-1}$
						$\frac{1}{x}$

write	$\frac{1}{3}x$	$\frac{1}{2}(a+b)$	$\frac{a+\frac{1}{2}b}{\frac{1}{3}c+\frac{1}{4}d}$	$\frac{a}{b+c/d}$	$\sqrt{x}$ or $x^{\frac{1}{2}}$	$i$ or $i$
						$x^{-1}$

instead of	$\frac{1}{x^n}$	$x \cdot \overline{x+a}$	$\sqrt{x-y}$	$e^{\frac{n\pi x}{a}}$	$\int_0^{\frac{\pi}{2}}$	$\lfloor n \rfloor$

write	$x^{-n}$	$x(x+a)$	$\sqrt{(x-y)}$ or $(x-y)^{\frac{1}{2}}$	$e^{n\pi x/a}$	$\int_0^{\frac{1}{2}\pi}$	$n!$

In current ordinary text—

instead of	$\frac{x}{a}$	$\frac{a+b}{c+d}$	$\frac{x}{y+\frac{t}{z}}$	$x/y + \frac{a}{b+c}$
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write	$x/a$	$(a+b)/(c+d)$	$x/(y+\frac{1}{2}t)$	$\frac{x}{y} + \frac{a}{b+c}$
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Excessive use of the slanting line, or solidus, is, however, undesirable; it may often be avoided by placing several short fractions or formulæ, with the intervening words if any, on the same line, instead of setting out each one on a line by itself. The last of the examples given above illustrates an improper use, in which symmetry is spoiled while nothing is gained; either both fractions should be written with the solidus, as  $x/y + a/(b+c)$ , or else neither as above.

The solidus should be of the same thickness as the horizontal line which it replaces; in some founts of type it is too thick and prominent.

Irregularities in the spacing of letters and symbols in the formulæ as printed are often the cause of a general unsatisfactory appearance of the page.

For centimetres, millimetres, kilometres, grammes, kilogrammes, the abbreviations should be cm., mm., km., gm., kgm. (not cms., etc.), and so in similar cases. Present custom is against the use of the signs  $\cdot\cdot$  and  $\cdot\cdot\cdot$ .

Symbols which are not provided in the usual founts of type are, as a rule, to be avoided. Compounded symbols such as  $\dot{a}$  or  $\ddot{a}$  usually involve justification, and are thus liable to become deranged or broken. The two examples here given have, however, become so essential that separate founts should be provided for them.

The use of a smaller fount for numerical fractions is now customary; thus always  $\frac{1}{3}a$  instead of  $a/3$ . The use of negative exponents often avoids a complex fractional form; as also the use of the fractional exponents, such as  $\frac{1}{2}$  and  $\frac{1}{3}$ . In the latter case  $x^{\frac{1}{3}}$  is usually preferred to  $x^{1/2}$ , notwithstanding that the latter is more legible.

Much is often gained in compactness and clearness by setting out two or more short formulæ on one line, instead of on consecutive lines; in that case they should be separated by spaces, indicated by the sign # on the MS. This would apply with even greater force to expressions such as  $x = a, = b, = c$ .

In the Preface to his *Mathematical and Physical Papers*, vol. i., 1880, the late Sir George Stokes successfully introduced the limited use of the solidus notation, obtaining the assent and support of Lord Kelvin, Prof. Clerk Maxwell, Lord Rayleigh, the Editors of the *Annalen der Physik*, and many other mathematicians. He defined its use as restricted to the symbols immediately on the two sides of it, unless a brace or stop intervenes; thus  $\sin n\pi x/a$  is to mean  $\sin(n\pi x/a)$ ; but  $\sin n\theta./r^n$ , in case it is used, would mean  $(\sin n\theta)/r^n$ .